

## Chapter 7. Present Value

**S7.1.** This problem is deals with both Present Value and Prepayment Penalties. Since Present Value is discussed in Chapter 7 while Prepayment Penalties are discussed in Chapter 5, I'm putting this problem as part of the Chapter 7 problem set so that all appropriate materials have been presented before you try to solve the problem.

You just took a \$100,000 loan at an APR of 6%, compounded annually. You will be paying this loan back with equal annual payments over the next 10 years. Your lender's actual cost for setting up this loan was \$2,500. He considers this \$1,000 one of his costs of doing business, and will be making enough profit from your payments to cover this cost.

a) If your lender has access to money at a 4% APR, what is the present value of his profit on day 1 of the loan?

Notes: (1) Although monthly interest compounding and payments are much more common than are annual ones, I'm using the latter so as to keep the resulting amortization tables short. (2) Don't be surprised that your lender can get money at a lower rate than you can; he might be a savings bank who is giving 4% interest on savings accounts, or a borrower himself from another bank who gets better rates than you can both because he's borrowing huge sums of money (and loaning pieces to many lenders) and because he's averaging his risks (e.g. of you going bankrupt) over many relatively small loans. (3) \$2,500 is a ridiculously large cost in this situation. I chose it so that the calculations based on it would be clear and not muddled up by sporadic rounding off of pennies. The calculations would be the same whatever the cost.

There are two loans to be considered here; your loan from your lender and your lender's loan from his lender. The spreadsheet Ch3Amortization.xls will handle these situations – just remember to multiply the Rate by 12 and treat monthly payments as annual payments.

First, your loan:

Pmt Nr	Balance	Payment
0	\$100,000.00	\$0.00
1	\$92,413.20	\$13,586.80
2	\$84,371.20	\$13,586.80
3	\$75,846.68	\$13,586.80
4	\$66,810.68	\$13,586.80
5	\$57,232.53	\$13,586.80
6	\$47,079.68	\$13,586.80
7	\$36,317.67	\$13,586.80
8	\$24,909.93	\$13,586.80
9	\$12,817.73	\$13,586.80

10	\$0.00	\$13,586.80
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Your lender must borrow \$102,500 so that he can loan you \$100,000 and pay the up-front \$2,500 costs:

Pmt Nr	Balance	Payment
0	\$102,500.00	\$0.00
1	\$93,962.68	\$12,637.32
2	\$85,083.86	\$12,637.32
3	\$75,849.90	\$12,637.32
4	\$66,246.57	\$12,637.32
5	\$56,259.11	\$12,637.32
6	\$45,872.15	\$12,637.32
7	\$35,069.72	\$12,637.32
8	\$23,835.19	\$12,637.32
9	\$12,151.27	\$12,637.32
10	\$0.00	\$12,637.32

Each month, your lender realizes a profit of the difference between your payment and his payment:  $\$13586.80 - \$12,637.32 = \$949.48$ . The present value of each of these payments at the beginning of the loan is calculated at his cost of money, 4%.

Calculating the present value of a set of equal payments made at periodic intervals is simple, even without a dedicated spreadsheet. The first payment is made 1 year after the beginning of the loan and has a present value at the beginning of the loan of  $\$949.48 / (1 + 0.04) = \$912.96$ . The present value of the second payment is just the present value of the first payment calculated one year later;  $\$912.96 / (1 + 0.04) = \$877.85$ . This process is continued for all ten payments:

Pmt Nr	Payment	Present Value
0	\$0.00	\$0.00
1	\$949.48	\$912.96
2	\$949.48	\$877.85
3	\$949.48	\$844.08
4	\$949.48	\$811.62
5	\$949.48	\$780.40
6	\$949.48	\$750.39
7	\$949.48	\$721.53

8	\$949.48	\$693.78
9	\$949.48	\$667.09
10	\$949.48	\$641.43

Finally, the total present value of your lender's profit on day one of (the beginning of) your loan is just the sum of all ten present values, = \$8,047.16.

**b)** If you immediately repaid the loan, not only would your lender not realize any profit, he would never have recouped his \$2,500 costs. It would therefore seem fair (at least to him) that in this situation he should be able to charge you a \$2,500 prepayment penalty. Assuming that the lender is amortizing the \$2,500 cost as part of his loan, what would the prepayment penalty be at the beginning of each of the ten years of the loan?

We could answer this question “starting from scratch” or more easily answer it by just scaling the lender's amortization table that was already created. \$2,500 is 2.44% of \$102,500. After one year, therefore, the remaining “debt” due to the \$2,500 cost is  $.0244(\$93,962.68) = \$2291.78$ . After two years it is  $.0244(\$85,083.86) = \$2,050.86$ , and so on. Note that present value calculations are not needed because a prepayment penalty would be due at the time of the repayment of the loan. The present value of these amounts at the beginning of the loan could of course be calculated, but are of no interest to us.