Chapter 10. Life Insurance

S10.1 Manufacturers of products such as toasters and TV sets usually sell their products with a warranty. If the quality of the product is so high that very few products are ever returned, it doesn't pay for the manufacturer to actually maintain service shops. The overhead (equipment, space, salaries) exceeds the cost of simply taking back the bad unit and sending out a new unit as a replacement[1]. A five year warranty on a product may be thought of as a five year term life insurance policy accompanying the product with the amount of insurance equaling the replacement cost of the product.

Assume the product failure statistics for a certain TV set are as shown in the following table. The definitions of variables x and q are the same as in the Life Tables presented in Chapter 10. In this case “death” is equivalent to “defective product return.”

Age (x)	**q**
0	1.6×10^{-5}
1	2.1×10^{-6}
2	3.0×10^{-6}
3	4.0×10^{-6}
4	5.0×10^{-6}

Aside: The q values above are so small that it made sense to write them in “scientific notation.” In this notation, the exponent after the 10 tells you how many decimal places to move the “.” in the number preceding the 10. A negative exponent, as above, means “move the decimal place to the left,” a positive exponent means “move the decimal place to the right.” For example $2.1 \times 10^{-6} = 0.0000021$. Since $10^{-6} = 1$ million**th,** 0.0000021 is often called “2.1 parts per million.”

For every million \$500 TVs sold in a year, what is the manufacturer's expected warranty cost? For this problem, neglect overhead cost details and just assume the it costs the manufacturer \$500 to replace a defective TV. Assume that the sets are all sold, and those that break all break, at the middle of the year. Assume that the TV company borrows money at an 8% APR.

Another Aside: When working with the actual Life Tables, q values are of the order of 0.01. This means that if there are 100,000 people alive at the beginning of the year, approximately 1,000 people will die during the year. Stating this a bit differently, we started with about 100 times the number of deaths we expected during the year. This let us comfortably round the number of deaths to integer values without generating nonsensical results. When dealing with probabilities of the order of 10^{-6}, therefore, we should start with 10^{8} (100 million) “live” TV sets at the beginning of the year. In terms of the actual statistics of a situation where the manufacturer is only selling about 1 million TV sets each year this means that there will be a large variation, or standard deviation, about the expected value. Delving deeper into this issue requires an in-depth look at the subject of quality control statistics, which is very far from our topic base. We'll limit our study to the expected values.

1 This doesn't mean that the returned, defective, units are just trashed. They are analyzed very carefully so that the cause of the defect can be “designed out” of the next generation of products.

x	q	l	d	cost	PV
0	$1.6x10^{-5}$	100,000,000	1,600	$800,000	$800,000
1	$2.1x10^{-6}$	99,998,400	210	$104,998	$97,221
2	$3.0x10^{-6}$	99,998,190	300	$149,997	$128,598
3	$4.0x10^{-6}$	99,997,890	400	$199,996	$158,763
4	$5.0x10^{-6}$	99,997,490	500	$249,994	$183,753

The warranty cost is calculated exactly the same as you would calculate the up-front premium for a multi-year term life policy except that the present value is calculated slightly differently because the problem specified that everything be referenced to the middle of the first year (x = 0). The total cost at that time is just the sum of the individual present values, $1,368,335. This seems like a lot of money until you calculate that this is just 0.27% of the sales dollars for the year. This translates to $1.35 warranty cost per TV sold.

S10.2 Some products, such as automobile batteries are sold with a "prorated warranty." For example, for the first year after sale the warranty is for 100%, for the second year it is 80%, etc. Using the same q values and all other assumptions as above, what is the warranty cost per $100 battery sold?

The easiest way to solve this problem is to scale the above example, get the percent cost per unit sold, and then multiply this by $100:

x	q	l	d	cost	PV	proration	Prorated PV
0	$1.6x10^{-5}$	100,000,000	1,600	$800,000	$800,000	100%	$800,000
1	$2.1x10^{-6}$	99,998,400	210	$104,998	$97,221	80%	$77,777
2	$3.0x10^{-6}$	99,998,190	300	$149,997	$128,598	60%	$77,159
3	$4.0x10^{-6}$	99,997,890	400	$199,996	$158,763	40%	$63,505
4	$5.0x10^{-6}$	99,997,490	500	$249,994	$183,753	20%	$36,751

The sum of the prorated present values is $1,055,191 which is 77% of the sum in problem S10.1. The cost of warranty per battery is therefore (.77)(.0027) = 0.21% of the battery cost, or $0.21 for each battery sold.